4-6 Cramer's Rule

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- used determinants to solve systems of equations

Use elimination to solve for x:

$d(ax + by = e)$

$-b(cx + dy = f)$

$adx + bdy = de$

$-bcx - bdy = -bf$

$(ad - bc)x = de - bf$

$x = \frac{de - bf}{ad - bc}$

$ax + by = e$

$cx + dy = f$

$x = \frac{de - bf}{ad - bc}$

Denominator is:

$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

Write a determinant matrix for the numerator:

$\begin{vmatrix} e & b \\ f & d \end{vmatrix}$

Cramer's Rule:

$x = \frac{D_x}{D} \quad y = \frac{D_y}{D}$

Where D_x replaces the coefficients of x with the constants and D_y replaces the coefficients of y with the constants.

ex:
x - 3y = 5
2x + 9y = -10

$x = \frac{D_x}{D}$ $\frac{15}{15} = 1$

$D = \begin{vmatrix} 1 & -3 \\ 2 & 9 \end{vmatrix} = 15$

$D_x = \begin{vmatrix} 5 & -3 \\ -10 & 9 \end{vmatrix} = 15$ (30)

$D_y = \begin{vmatrix} 1 & 5 \\ 2 & -10 \end{vmatrix} = -20$ (+10, 45, -10)

$y = \frac{D_y}{D}$ $\frac{-4}{1}$

$(1, -4, 1, 3)$

If D = 0

then, either

∞ # of solutions

OR

∅

ex:
2x - y = 3
6x - 3y = -9

(-3) $-6x + 3 = -9$
$6x - 3 = -9$
$0 = -18$

$D = \begin{vmatrix} 2 & -1 \\ 6 & -3 \end{vmatrix} = 0$

$D_x = \begin{vmatrix} 3 & -1 \\ -9 & -3 \end{vmatrix}$

$D_y = \begin{vmatrix} 2 & 3 \\ 6 & -9 \end{vmatrix}$

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3 Variables

Cramer's Rule:

$x = \frac{D_x}{D}$ $y = \frac{D_y}{D}$ $z = \frac{D_z}{D}$

ex:
4x + y - z = -2
x + 3y - 4z = 1
2x - y + 3z = 4

$$D = \begin{vmatrix} 4 & 1 & -1 \\ 1 & 3 & -4 \\ 2 & -1 & 3 \end{vmatrix} = 16$$

$$D_x = \begin{vmatrix} -2 & 1 & -1 \\ 1 & 3 & -4 \\ 4 & -1 & 3 \end{vmatrix} = -16$$

$$D_y = \begin{vmatrix} 4 & -2 & -1 \\ 1 & 1 & -4 \\ 2 & 4 & 3 \end{vmatrix} = 96$$

$$D_z = \begin{vmatrix} 4 & 1 & -2 \\ 1 & 3 & 1 \\ 2 & -1 & 4 \end{vmatrix} = 64$$

$x = \frac{-16}{16} = -1$ $y = \frac{96}{16} = 6$ $z = \frac{64}{16} = 4$

HW
p 192-193
12, 13, 26-28, 30